MODELS OF HEAT TRANSFER OF A COOLED ac CONDUCTOR

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Models are proposed for the heat transfer of an ac conductor cooled by a cryogenic liquid. On the basis of numerical solutions, various thermal situations are analyzed.

The heat transfer at cryogenic temperatures of a cooled dc conductor was considered in detail in [1], in which thermal phenomena in such cryogenic systems were discussed and recommendations for the calculation and thermal stabilization of such systems were given. The thermal state of a stabilized ac superconductor cooled by a cryogenic liquid was first investigated in [2-4] from the viewpoint of emergencies in such systems and thermal stability. However, there was no detailed discussion of the accurate description of nonsteady heat transfer, and the steady heat-transfer coefficient was used everywhere in the calculations.

In the present work, the nonsteady thermal processes in a homogeneous ac conductor immersed in a bath of cryogenic liquid (helium) are considered. Models are proposed for the description of nonsteady heat transfer in the system in the case of currents of low and high frequency, and the limits of their applicability are indicated. Numerical implementation of the models permits a detailed analysis of the nonsteady thermal field and various situations – in particular, overheating of the conductor – taking into account the real nonlinear temperature dependences of the specific heat and resistivity of the conductor.

The conductor is assumed either to be hollow, with a wall thickness of fractions of millimeter, or else to consist of thin strands, so that the skin effect may be neglected even at the lowest temperatures. Since the conductor is homogeneous and of sufficiently large thermal conductivity, the temperature difference along the conductor is neglected, as is its variation over the cross section, which estimates show to be acceptable.

Under these assumptions, the thermal state of the conductor is described by the nonlinear nonsteady thermal-balance equation

$$C(T)S\frac{dT}{dt}=\frac{I^2(t)\rho(T)}{S}-q(t)\Pi. \tag{1}$$

Here ρ is the resistivity of the conductor; C is its specific heat; Π and S are, respectively, its perimeter and cross section; I(t) is the current flowing along the conductor; and q is the heat flux per unit surface of the conductor in the cooling liquid.

To simplify the discussion, the origin of the time coordinate is shifted, and the basic frequency ω of the process is taken to be twice the current frequency, so that it is written in the form $I(t)=i\sqrt{2}\sin[(\omega/2)t+(\pi/4)]$, where is is a virtual value. The temperature scale is also shifted, so that the temperature of the coolant liquid at infinity is zero. Scaling C and ρ with respect to their values at some characteristic temperature T_0, the time with respect to $1/\omega$, and the heat flux q with respect to $\alpha_0 T_0$, corresponding to heat transfer from the level T_0 with a steady coefficient α, Eq. (1) may be brought to dimensionless form (the primes on the dimensionless quantities are omitted):

$$\Omega C(T)\frac{dT}{dt}=\varkappa(\rho(T)+\rho(T)\sin t)-q(t). \tag{2}$$

Two dimensionless complexes determine the thermal conditions in the problem: $\varkappa=i^2\rho_0/\alpha_0\Pi ST_0$ which depends on the current i and is an analog of the Steckley parameter for stabilized superconductors, and $\Omega=C_0\omega S/\alpha_0\Pi$, characterizing the heat-liberation frequency ω.

It is simple to estimate the temperature pulsations of the conductor under the assumption that the nonsteady heat flux q(t) is of the same order of magnitude as the steady expression $\alpha(T)T$. Assuming that the temperature oscillates with the small amplitude δT about some value $\langle T\rangle \gg \delta T$, the linearized relation (2) yields a solution of the form

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$$T = \langle T \rangle - \delta T \cos t, \tag{3}$$

where $\langle T \rangle = \varkappa \dfrac{\rho(\langle T \rangle)}{\alpha(\langle T \rangle)} \sim \varkappa$, and $\delta T = \dfrac{\varkappa}{\Omega} \cdot \dfrac{\rho(\langle T \rangle)}{C(\langle T \rangle)} \sim \dfrac{\varkappa}{\Omega}$, so that $\delta T/\langle T \rangle \sim 1/\Omega$. Hence, it is only correct to assume that the pulsations are small when $\Omega \gg 1$. Estimates show that even for a continuous copper conductor of diameter 5 mm at a current frequency of 50 Hz, with film boiling of the helium, the parameter Ω is close to unity. The parameter $\varkappa$ is also of order unity in real power systems [1]. Thus, the temperature of the given conductor oscillates with considerable amplitude. This means that accurate solution of the nonlinear problem is necessary, and, hence, the description of the nonsteady heat flux q(t).

Two limiting cases are possible. At sufficiently small frequencies, the system is able, as it adjusts at each moment of time, to respond to the whole temperature variation of the conductor. In this case, it is possible to use the quasisteady approximation

$$q = \alpha(T)\,T. \tag{4}$$

The heat-transfer coefficient $\alpha(T)$ is the same as in the steady case, and all the time variation is included in T(t).

At sufficiently high frequencies, the coolant liquid cannot respond, in the hydrodynamic sense, to the temperature pulsations of the conductor wall: No new vapor bubbles form in the case of boiling, appropriate adjustments to the flow are impossible in the case of single-phase convection, etc. The thermal perturbations are transmitted in the liquid solely by heat conduction. Therefore, in this case the thermal flux q(t) consists of a steady flux with a mean temperature level $q_{st} = \alpha(\langle T \rangle)\langle T \rangle$ and a nonsteady increment δq_{nonst} due to heat conduction. To determine this increment, the thermal boundary layer around the conductor will be taken to be plane. The linear heat-conduction equation $\partial T/\partial t = a\,\partial^2 T/\partial x^2$, written in dimensionless variables, yields a solution dying out along the x axis in the liquid:

$$T(x,\ t) = T^0 \exp\left(-\sqrt{\frac{\omega}{2a_c}}\,x\right)\sin\left(\omega t - \sqrt{\frac{\omega}{2a_c}}\,x\right) + \langle T \rangle, \tag{5}$$

where a_c is the thermal diffusivity of the coolant and T^0 is the amplitude of the temperature oscillations at the wall. The depth of penetration of the thermal perturbations into the liquid falls as $\sqrt{a_c/\omega}$ with increase in frequency. For liquid helium in the case of film boiling at a frequency of 50 Hz the penetration depth is about 0.012 mm, and in the case of convection in supercritical helium it is about 0.007 mm, which justifies the assumption that the boundary layer is plane. The dimensionless steady heat flux in the coolant corresponding to Eq. (5) is

$$q = \eta\,\frac{T^0}{T_0}\sin\left(t + \frac{\pi}{4}\right), \tag{6}$$

where $\eta = (\lambda_c/\alpha_0)\sqrt{(\omega/a_c)}$; λ_c is the thermal conductivity of the coolant.

In the problem under consideration, the dimensionless temperature of the conductor may be written as a Fourier series

$$T = \langle T \rangle + \sum_{n=1}^{\infty} T_n \sin(nt + \varphi_n) = \langle T \rangle + \sum_{n=1}^{\infty}(A_n \cdot \sin nt + B_n \cos nt). \tag{7}$$

Therefore, taking into account Eq. (6), the total heat flux in the liquid $q(t) = q_{st} + \delta q_{nonst}$ is the given model is

$$q(t) = \alpha(\langle T \rangle)\langle T \rangle + \eta \sum_{n=1}^{\infty} \sqrt{n}\, T_n \sin\left(nt + \varphi_n + \frac{\pi}{4}\right). \tag{8}$$

In the case of free convection this approach corresponds to the Lighthill approximation [5] and holds as long as the amplitude T_n is not too large.

To solve the problem in the case of large frequencies, Eq. (8) must be substituted into Eq. (2). After equating corresponding harmonics in Eq. (2), an infinite algebraic system of equations in the coefficients A_n and B_n is obtained; this system may be solved numerically if a finite number of harmonics T_n is taken. To obtain the solution, the harmonic functions ρ(t) and C(T) must be expressed in terms of A_n and B_n. At helium temperatures the dependence of the resistance and the specific heat on the absolute temperature may be written in the form [6]

$$\rho(T) = \rho_{res} + \rho_i T^5,\ C(T) = C_e T + C_i T^3. \tag{9}$$

Tedious calculations lead to the expressions

$$T^3 = \frac{1}{4} \sum_{\beta,\gamma=0}^{1} \sum_{i,j,k=0}^{\infty} \{(3A_iB_jB_k - (-1)^{\beta+\gamma} A_iA_jA_k) \sin [i + (-1)^{\beta} j + (-1)^{\gamma} k]\, t + (B_iB_jB_k - 3(-1)^{\beta} A_iA_jB_k) \cos [i + (-1)^{\beta} j + (-1)^{\gamma} k]\, t\},$$

$$T^5 = \frac{1}{16} \sum_{\beta,\gamma,\delta,\varepsilon=0}^{1} \sum_{i,j,k,l,m=0}^{\infty} \{((-1)^{\beta+\gamma+\delta+\varepsilon} A_iA_jA_kA_lA_m - 10(-1)^{\beta+\gamma} \times A_iA_jA_kB_lB_m + 5A_iB_jB_kB_lB_m) \sin [i + (-1)^{\beta} j + (-1)^{\gamma} k + (-1)^{\delta} l + (-1)^{\varepsilon} m]\, t + (B_iB_jB_kB_lB_m - 10(-1)^{\beta} A_iA_jB_kB_lB_m + 5(-1)^{\beta+\gamma+\delta} A_iA_jA_kA_lB_m) \cos [i + (-1)^{\beta} j + (-1)^{\gamma} k + (-1)^{\delta} l + (-1)^{\varepsilon} m]\, t\},$$

which are then used in Eq. (9) to solve the given model.

Consideration of Eq. (8) indicates that the transition frequency from the quasisteady description in Eq. (4) to the description in Eq. (8) is characterized by the characteristic times of the elementary heat-transfer processes

$$\omega' = 2\pi/\tau_{\text{char}}. \tag{10}$$

In the case of the bubble boiling of helium, τ_{char} is the time of bubble growth, equal to about 0.03 sec according to estimates using the formulas of [7]; in the case of laminar convection in supercritical helium it is the time for adjustment of the convective fluxes, which is found to be about 0.15 sec in an estimate according to [8, 9].

As well as Eq. (10), there is one other constraint on the use of the quasisteady approximation; this arises because it is not possible, in the steady heat-transfer coefficient α, to take account of the correction due to nonsteady molecular heat conduction, increasing as $\sqrt{\omega}$ with increase in frequency in accordance with Eq. (6). For the quasisteady approximation to be valid, it is necessary for the frequency to be less than

$$\omega'' = a_c\, \alpha_0^2/\lambda_c^2. \tag{11}$$

The applicability of the nonsteady model (8) is also associated with Eq. (11); it is only for frequencies not less than ω'' in order of magnitude that the value of η in Eq. (6) is not too small, and the amplitude of the temperature pulsations is significantly less than the mean value of the temperature, which must be the case if Eq. (8) is to be used. Both for film boiling of helium and for convection in supercritical helium, Eq. (11) is satisfied at current frequencies of about 50 Hz.

From the foregoing it follows that to calculate transient processes with frequencies of the order of 1 Hz the quasisteady approximation may be used, while for operating conditions of the system with a current frequency of 50 Hz, Eq. (8) may be used.

Numerical calculations of both models have been made on a BÉSM-6 computer.

In the quasisteady approximation Eq. (2), together with Eq. (4), was solved by the Runge–Kutta method. The function $\alpha(T)$ was approximated by the dependence $T^{0.25}$, which is acceptable for film boiling and free convection [4]. The steady solution was expanded in Fourier series.

In the large-frequency case, the system of nonlinear algebraic equations in A_n and B_n was solved by the optimum-search method. Three harmonics were found to be sufficient; the error due to discarding the other terms was then satisfactorily small, since numerical calculation shows that further increase in the number of harmonics does not lead to perceptible change in the results. The coefficient η in Eq. (6) was written in the form $\eta = \eta_1\sqrt{\Omega}$, where η_1 was taken equal to 0.1, 0.3, and 1; this corresponds to film boiling for a conductor of diameter 10 cm, 1 cm, and 1 mm and for convection in supercritical helium for a conductor of diameter about 20 cm, 2 cm, and 2 mm, respectively.

The results show that steady conditions set in some time after the beginning of the process, and this time increases with increase in Ω. Note that for both models the mean temperature $\langle T \rangle$ of the conductor, increasing with increase in the current, is largely independent of change in Ω even up to the highest frequencies. Therefore, to determine $\langle T \rangle$ approximately, it is in most cases sufficient, in practice, using the analogy with the dc case, to solve the equation $\varkappa\rho(\langle T \rangle) = \alpha(\langle T \rangle)\langle T \rangle$.

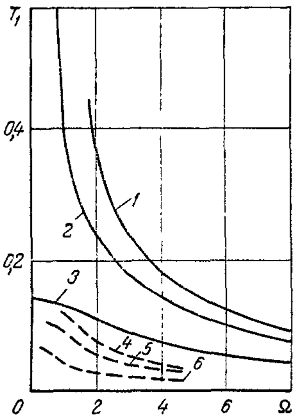

Fig. 1. Amplitude of first harmonic temperature pulsations of conductor. Curves 1-3 correspond to the quasisteady approximation: $\varkappa = 1.0$ (1), 0.8 (2), and 0.4 (3). Curves 4-6 correspond to the high-frequency approximation (8) $\varkappa = 0.4$; $\eta_1 = 0.1$ (4), 0.3 (5), and 1.0 (6).

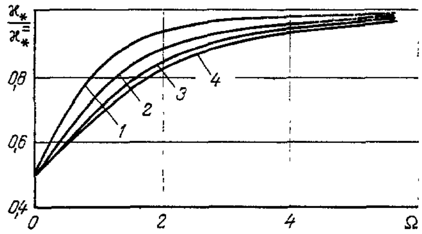

Fig. 2

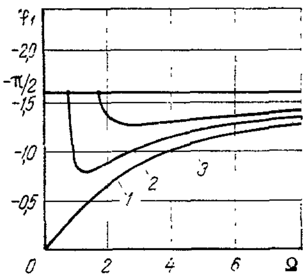

Fig. 3

Fig. 2. The dependence of $\varkappa_*/\varkappa_*^{=}$ on Ω: 1) $R_0 = 0.2$; 2) 0.5; 3) 0.8; 4) 1.0.

Fig. 3. Dependence of phase of first temperature harmonic on Ω ($G_0 = 0.7$, $R_0 = 0.8$): 1) $\varkappa = 0.4$; 2) 0.8; 3) 1.0 (quasisteady approximation).

The dependence on Ω of the amplitude of the first and higher harmonics is shown in Fig. 1. It is evident that in both models the amplitude of the pulsations falls with increase in frequency, the pulsations being somewhat smaller in the high-frequency approximation than in the quasisteady case.

Steady thermal conditions are observed as long as the current does not exceed some critical value specific to each Ω [3]. If the conductor is loaded by a current above this value, the temperature begins to pulsate and to increase catastrophically, and the conductor may burn out. This overheating is associated with the nonlinear dependence $\rho(T)$, since in the dc case [1], at sufficiently high I, the heat-liberation curve $I^2\rho(T)/S$ is everywhere above the heat-extraction curve $\alpha\Pi T$, heat liberation always predominates over heat extraction, and no stable state of the conductor exists. For the ac case the situation is analogous [2-4]. The effective value of the current at which the stable state existing at lower currents disappears will be called the overheating current i_*. Under ac conditions, overheating sets in at currents less than the value $i_*^{=}$ corresponding to the dc case; as shown in [3], the ratio $\varkappa_*/\varkappa_*^{=}$ varies from 0.5 as $\Omega \to 0$ (because of the lack of thermal inertia, the amplitude value of the current $i_*\sqrt{2}$ coincides with $i_*^{=}$) to unity as $\Omega \to \infty$. The precise course of the curve of $\varkappa_*/\varkappa_*^{=}$ as a function of Ω depends on the relations between the coefficients in Eq. (9), i.e., on the conductor material. The ratios of the last terms in Eq. (9) to the total values of ρ and C at some characteristic temperature will be denoted by R_0 and G_0, respectively. The R_0 characterizes the frequency of the conductor [6]: As the impurity is decreased, so too the contribution of the residual resistance ρ_{res} diminishes, and R_0 approaches unity. The dependence of $\varkappa_*/\varkappa_*^{=}$ on Ω is shown in Fig. 2 for different values of R_0 but for the constant value $G_0 = 0.7$. It is evident that decrease in R_0 is associated with decrease in the frequency at which the analogy with the dc case becomes applicable. Less pure materials give a smaller error in calculating the overheating by the simple dc formulas.

Consideration of the phase of the first temperature harmonic under steady conditions leads to another interesting conclusion (Fig. 3). When the current does not exceed the critical value i_* for any frequency (curve 1), the phase changes from zero at small frequencies (the system is warmed through) to $-\pi/2$ as $\Omega \to \infty$, since Eq. (3) is valid in this region. If the current exceeds the critical value at some frequency (curves 2 and 3), then in the region close to overheating ($i < i_*$) the phase decreases, tending to $-\pi/2$ in the immediate vicinity of overheating. (Note that the time required to establish thermal equilibrium increases with approach to overheating.) This behavior of the phase is observed in both models for any values of R_0 and G_0. Analytical consideration at large Ω also confirms this result: Everywhere close to overheating $\varphi_1 \to -\pi/2$. Hence, the phase of the temperature just before overheating may be judged from the extent to which the current approaches the danger value i_*.

NOTATION

t, time; T, temperature; T_n, φ_n, amplitude and phase of the n-th temperature harmonic of the conductor; I, i, instantaneous and effective values of the current; ω, angular frequency of heat liberation; ρ, resistivity; C, specific heat of conductor; Π, perimeter; S, cross-sectional area of conductor; q, heat flux; α, steady heat-transfer coefficient; a_c, λ_c, thermal diffusivity and thermal conductivity of coolant; τ_{char}, characteristic time of elementary heat-liberation process; R_0, G_0, dimensionless parameters characterizing the form of the functions $\rho(T)$ and $C(T)$; $\varkappa$, Ω, η, η_1, dimensionless parameters; i_*, effective value of ac current corresponding to overheating of the conductor; $i_*^=$, dc current corresponding to overheating.

LITERATURE CITED

1. V. A. Al'tov, V. B. Zenkevich, M. G. Kremlev, and V. V. Sychev, Stabilization of Superconducting Magnetic Systems [in Russian], Énergiya, Moscow (1975).
2. V. A. Altov [Al'tov], N. A. Kulysov, and V. V. Sychev, Advances in Cryogenic Engineering, Vol. 22, Plenum, New York–London (1977).
3. V. A. Al'tov, N. A. Kulysov, and V. V. Sychev, in: Abstracts of Papers at the All-Union Conference on Technical Uses of Superconductivity, Alushta, 1975 [in Russian], Informélektro (1975), p. 31.
4. V. A. Altov [Al'tov], N. A. Kulysov, and V. V. Sychev, "Some results of investigation of composite conductors under ac conditions," Paper at the Fourth Soviet–American Symposium on the Technology of Superconducting Transmission Lines, Brookhaven (1976)..
5. R. S. Nanda and V. P. Sharma, J. Fluid Mech., 15, 419 (1963).
6. M. P. Malkov (editor), Handbook on Physical and Engineering Cryogenic Principles [in Russian], Énergiya, Moscow (1973).
7. S. S. Kutateladze, Principles of Heat-Transfer Theory [in Russian], Nauka, Novosibirsk (1970).
8. A. J. Eade, "Free convection," in: Achievements in Heat Transfer [Russian translation], Mir, Moscow (1970).
9. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1967).